On approaching wideband capacity using multi-tone FSK

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Abstract

In the wideband limit, certain types of "flash" signaling, such as flash Frequency-Shift Keying (FSK), achieve the capacity of multipath fading channels. It is not clear, however, whether these asymptotic results translate into insights for practical fading channels in the finite-bandwidth, power-limited regime. It is known that, for flash FSK, the size of the input alphabet grows slowly with increasing bandwidth, leading to very high peak power per tone. Thus, for flash FSK, the codeword probability of error decays very slowly with bandwidth and feasible rates approach the wideband capacity limit extremely slowly. Without contradicting the above results, our results in this paper point to a more optimistic outlook, from the point of view of error exponents and achievable rates, for the applicability of flash techniques in practical scenarios. We consider multi-tone FSK (MFSK), which has the same asymptotic capacity-achieving property as flash FSK in the wideband limit, but allows a larger input alphabet size with the same bandwidth. First, we show, using an error exponent approach, that multi-tone FSK allows lower peak power per tone than flash FSK. Next, we present the capacity of single-tone and two-tone FSK schemes with hard decision detection at finite bandwidths. For typical channel parameters, the capacities are close to the wideband capacity limit.

Index Terms

Frequency shift keying, information rates, communication system performance, mobile communication, broadband communication

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I. Introduction

The study of communication in wideband fading channels dates back to the 1960's. In [1], Kennedy has shown that, for a given average received power, the capacity of a Rayleigh fading channel has the same limit as that of an additive white Gaussian noise (AWGN) channel as the bandwidth approaches infinity. This result is true even if the receiver does not have any channel state information (CSI). Telatar and Tse [2] generalized this result and showed that Shannon's wideband capacity limit [3] is indeed achievable with any distribution of the channel fading coefficients.

The focus of our paper is on the non-coherent case, i.e., CSI is not known at either the transmitter or the receiver. In this case, as the average signal energy in each coherence bandwidth decreases, the cost of estimating the channel fading coefficients becomes more and more significant. While the capacity of non-coherent channels is the same as that of coherent channels in the wideband limit, the lack of CSI severely affects the performance of several commonly used signaling schemes in the wideband regime.

One example is the *bandwidth-scaled spread spectrum signaling* defined in [4]. When the bandwidth W increases, the energy and fourth moment of the signal in each fixed frequency band scale with $1/W$ and $1/W^2$, respectively. Roughly speaking, the signaling spreads the signal energy evenly over the available bandwidth. Such signals encompass Gaussian signals, which are optimal in the coherent case (perfect receiver CSI). However, although bandwidth-scaled signaling has a good performance in the coherent case, it performs poorly in non-coherent scenarios. It has been shown in [4], [5] that, as the bandwidth increases, the achievable data rate using bandwidth-scaled signaling in fact decreases to 0. Therefore such signals, including most common DS-CDMA schemes, are not suitable for wideband transmission in frequency-selective fading channels.

The poor performance of bandwidth-scaled signaling schemes in wideband fading channels has led to alternative schemes. A commonly proposed scheme for ultra wideband indoor communication is pulse position modulation (PPM) and related schemes, often grouped under the ultra-wideband terminology. The data rates of these schemes are mainly constrained by the multipath spread $\mathcal{L}$ of the channel, defined as the difference in propagation time between the longest and shortest path. In particular, the symbol rate of PPM is bounded by $1/\mathcal{L}$. Thus, these schemes are suitable for indoor communications, where $\mathcal{L}$ is relatively small and stable. However, for wideband outdoor channels, $\mathcal{L}$ may be large and unstable, which significantly affects achievable rates.

A signaling, used in [2] [6, §8.6] to achieve the capacity of fading channels at the infinite bandwidth limit, is a special *impulsive*, or "flash", FSK scheme, which transmits FSK signals with a low duty cycle and a high peak power. Unlike the bandwidth-scaled signaling schemes, the flash FSK signaling does not spreads the signal energy evenly over the available bandwidth. Instead, its signal energy is peaky both in time and frequency. Because each symbol in the scheme has only one frequency, we call the scheme *single-tone FSK*, compared to the multi-tone FSK scheme that we will discuss later.

In [7], Verdú has shown that, in order to achieve the capacity of a wideband non-coherent fading channel, signaling must be peaky. In particular, let us consider, as in [7], the Taylor series expansion of capacity at vanishing signal-to-noise ratio (SNR) per degree of freedom, corresponding to the wideband scenarios,

$$C(\mathsf{SNR}) = \dot{C}(0)\mathsf{SNR} + \ddot{C}(0)\mathsf{SNR}^2 + o(\ \mathsf{SNR}^2),$$

where SNR is the SNR per degree of freedom, $\dot{C}(0)$ and $\ddot{C}(0)$ are the first and the second derivatives of the function $\mathsf{SNR} \mapsto C(\mathsf{SNR})$ at $\mathsf{SNR} = 0$. For a general class of channels, when CSI is not available at the receiver, flash signals are necessary in order to achieve the first order optimality, $\dot{C}(0)\mathsf{SNR} = \mathsf{SNR}$, which is Shannon's wideband limit for coherent channels.

A number of flash signaling schemes are first order optimal. They achieve essentially the same capacity limit as the bandwidth approaches infinity. However, if one is interested in the performance at a large but finite bandwidth, it is not clear whether or not the above asymptotic analysis provides a close approximation. In fact, different signaling schemes may have different constraints at a finite bandwidth.

For single-tone FSK, the data rate at a finite bandwidth, or equivalently as the SNR per degree of freedom increases from 0, is quickly limited by $\ln M$ (nats/symbol), where M is the size of alphabet, i.e., the number of total frequency points. Some results concerning the practicality of single-tone FSK in finite bandwidths are even more discouraging. From a peak power perspective, Lun et al. [8] [9] have shown that codeword error probability of the single-tone FSK scheme decreases roughly inversely with bandwidth, leading to the need for very high peak power tones for a small codeword error probability. From a capacity perspective, Verdú [7] has shown that, for flash schemes, the second derivative in the Taylor series expansion of capacity goes to $-\infty$ as $\mathsf{SNR} \to 0$, so that the wideband capacity limit is approached extremely slowly as the bandwidth increases.

The multi-tone FSK scheme discussed in this paper can achieve the infinite bandwidth capacity limit, and more importantly, it has several advantages over single-tone FSK which enable it to partially overcome the disadvantages of the single-tone FSK scheme.

For the same bandwidth, the number of symbols in MFSK is more than that in the single-tone FSK scheme. Thus, at a finite bandwidth, MFSK can do better if the data rate is mainly limited by the size of alphabet. For the same received signal power, MFSK has lower peak power per tone. Therefore, the peak power constraint per tone is less crucial for MFSK.

The goal of this paper is to characterize the performance of MFSK at both the wideband limit and a finite bandwidth.

- We generalize the results of [10] to derive an upper bound and a lower bound of the symbol error probability for MFSK. These two bounds lead to the error exponent of MFSK, which depicts the effect of bandwidth, data rate, number of tones, and duty cycle of the scheme on the error probability. In particular, we illustrate the fact that the use of MFSK can alleviate the very high per tone peak power required for the single-tone FSK.
- We present numerical results to show that simple single- and multi-tone FSK schemes with hard decision can yield rates approaching closely the wideband capacity limit at large but finite bandwidths, although we know that attaining rates arbitrarily close to capacity remains elusive.

The capacity of certain FSK schemes has been studied in other contexts. In [11], Stark determined the capacity of FSK schemes under non-selective Rician fading with receiver side information. In [12], non-coherent FSK was considered for Rayleigh fading channels with erasures. We consider instead a frequency selective block fading model with no CSI.

The rest of this paper is organized as follows: in Section II, we introduce our channel model and the multi-tone FSK scheme; in Section III, we calculate the bounds of error probability, and derive the capacity limit and the error exponent for multi-tone FSK; in Section IV, we present numerical results to show the performance of single- and multi-tone FSK at finite bandwidths; Section V contains our conclusions.

II. System Model

A. Frequency-selective block fading channel

Consider a multipath fading channel with input $x(t)$. The output $y(t)$ is given by

$$y(t) = \sum_{l=1}^{L} a_l(t)x(t - d_l(t)) + z(t), \tag{1}$$

where L is the total number of paths, the random processes $a_l(t)$ and $d_l(t)$ are the gain and the delay of the lth path, and $z(t)$ is complex white Gaussian noise with power spectral density $N_0/2$.

Wireless channels change both in time and frequency. The time coherence, $\mathcal{T}_c$, shows us how quickly the channel changes in time, and similarly, the frequency coherence, $\mathcal{F}_c$, shows how quickly it changes in frequency. The selective fading in frequency is caused by the different delays of path. The frequency coherence is reciprocal to the multipath spread, $\mathcal{L}$. The major effect in determining time coherence is the Doppler shift, $\mathcal{D}$, which causes significant changes in channel gain. The relationship between the time coherence and the Doppler shift is reciprocal.

Assuming that the multipath spread $\mathcal{L}$ is much less than the time coherence $\mathcal{T}_c$ (an under-spread channel), and the gain and the delay of a path are constant within each coherence block and change independently from block to block (block-fading), we obtain a frequency-selective block fading channel model (see Figure 1).

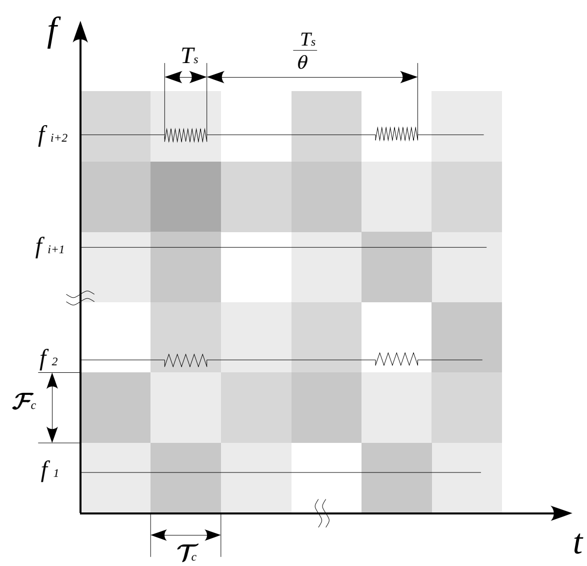

Fig. 1. Block fading channel demonstrated in a frequency-time plane

If two symbols are transmitted within the same square, they have the same fading coefficient; otherwise, they experience independent fading.

B. Multi-tone FSK

In FSK systems, different symbols are represented by sinusoid waveforms tuned to different specific frequencies. For the MFSK scheme, we use different combinations of multiple frequencies as symbols. For example, from a pool of M tones, i.e., M mutually orthogonal frequencies over a symbol slot, we can use different combinations of Q tones to represent different symbols. We fix the number of concurrent tones that form a symbol. The scheme using Q tones is called the *Q-tone FSK scheme*. Particularly, when Q equals to 1, the scheme is single-tone FSK, the flash FSK scheme used in [2] and [8].

In the MFSK scheme, transmissions take place in a low duty cycle fashion. The transmitter concentrates power over a fraction θ ($0 < \theta \leq 1$) of time, and transmits on predetermined symbol slots. We refer to θ as the *duty cycle*. The main reason for using a low duty cycle is the fact that transmission with insufficient power over fading environments are not desirable, and that we are power-limited with respect to the number of degree of freedom we have. Therefore, we purposedly use a low proportion of our degrees of freedom for transmission, leading to a low duty cycle. A repetition code of length N ($N \geq 1$) is employed, i.e., each symbol will be repeated N times.

At the receiver side, we use a non-coherent receiver to demodulate and decode MFSK signals. The receiver employs a bank of matched filters with their central frequencies tuned to each of the M tones, respectively. The output of each matched filter is the correlation of its tuned frequency against the received MFSK signal. Since the M tones are mutually orthogonal over a symbol time, the output of a matched filter is proportional to the amplitude of the associated tone. Thus, we can view the M tones as M frequency-division subchannels. For each subchannel, the time average of received power over the N consecutive symbol slots is obtained to compare with a threshold. If exactly Q tones exceed the threshold, then the corresponding symbol is decoded; otherwise, the receiver declares an error.

Note that, we do not seek an optimal detection/decoding rule here, which may lead to a better performance but more elusive analysis. Our goal is to provide a practical scheme, which achieves capacity in the wideband regime.

C. Mathematical model

For Q-tone FSK, since there are $\binom{M}{Q}$ possible Q-tone combinations from M tones, the number of total symbols is $\binom{M}{Q}$. We let $\mathcal{S}$ denote the complete set of symbols, and $\mathcal{S}_m$ a symbol in the set, i.e., $\mathcal{S}_m \in \mathcal{S}$. The fact that the kth tone is non-zero in a symbol $\mathcal{S}_m$ is denoted as $k \in \mathcal{S}_m$.

Assume the symbol time is T_s, which satisfies $\mathcal{L} \ll T_s \leq \mathcal{T}_c$ (this is possible because of the underspread assumption). To send a symbol $\mathcal{S}_m$, $x(t)$ is given by

$$x(t) = \sum_{k\in\mathcal{S}_m} \exp(j2\pi f_k t), \qquad 0 \leq t \leq T_s, \tag{2}$$

where $j = \sqrt{-1}$ and $\{f_k\}$ are M frequencies orthogonal over $[\mathcal{L}, T_s]$. Let T'_s denote $T_s - \mathcal{L}$. For a system bandwidth W, there exist $M = WT'_s$ frequencies mutually orthogonal over $[\mathcal{L}, T_s]$ which are integer multiples of $1/T'_s$.

Let us consider the channel output over the interval $[\mathcal{L}, T_s]$. Owing to the block fading channel assumption, the gains $\{a_{l,k}(t)\}$ and the delays $\{d_{l,k}(t)\}$ are constant during this interval, denoted as $a_{l,k}$ and $d_{l,k}$, where the subscripts, l and k, are the indices of path and tone, respectively. Hence, by (1), the received signal over $[\mathcal{L}, T_s]$ is

$$\begin{aligned} y(t) &= \sum_{k\in\mathcal{S}_m} \sum_{l=1}^{L} a_{l,k} \exp(j2\pi f_k(t - d_{l,k})) + z(t) \\ &= \sum_{k\in\mathcal{S}_m} g_k \sqrt{\frac{P}{Q\theta}} \exp(j2\pi f_k t) + z(t), \qquad \mathcal{L} \leq t \leq T_s, \end{aligned} \tag{3}$$

where P denotes the average received signal power, and g_k the aggregated channel gains for each tone,

$$g_k = \sum_{l=1}^{L} a_{l,k} \sqrt{\frac{Q\theta}{P}} \exp(-j2\pi f_k d_{l,k}).$$

We assume that the number of paths is very large and there is no dominant path. Based on this assumption, g_k are zero-mean complex Gaussian random variables according to the Central Limit Theorem. Their variances have been normalized, i.e., $E[|g_k|^2] = 1$, by putting the average received power term alone in (3).

At the receiver, the matched filter tuned to the kth tone outputs

$$y_k = \frac{1}{\sqrt{N_0 T'_s}} \int_{\mathcal{L}}^{T_s} \exp(-j2\pi f_k t) y(t) dt. \tag{4}$$

After manipulation, we obtain y_k,

$$y_k = \begin{cases} g_k \sqrt{\frac{PT_s'}{Q\theta N_0}} + w_k, & k \in \mathcal{S}_m, \\ w_k, & \text{otherwise;} \end{cases} \tag{5}$$

where the w_k, given by

$$w_k = \frac{1}{\sqrt{N_0 T_s'}} \int_{\mathcal{L}}^{T_s} \exp(-j2\pi f_k t) z(t) dt,$$

are mutually independent zero-mean complex Gaussian random variables with unity variance, because they are obtained by correlating white noise against orthogonal frequencies. With no requirement of phase information, the scheme is noncoherent. The phase error has already been considered in the calculation of the fading coefficients $\{g_k\}$.

According to (5), an equivalent channel can be obtained for the kth subchannel:

$$y_k = g_k x_k + w_k,$$

where the inputs x_k are given by

$$x_k = \begin{cases} \sqrt{\frac{PT_s'}{Q\theta N_0}}, & k \in \mathcal{S}_m, \\ 0, & \text{otherwise.} \end{cases}$$

As g_k and w_k are independent zero-mean complex Gaussian random variables, the equivalent channel is a Rayleigh fading channel.

For the Rayleigh channel, the transition probability between input x_k and sufficient statistic $r_k = |y_k|^2$ is

$$p_{r_k|x_k}(r|x) = \frac{1}{x^2+1} e^{-\frac{r}{x^2+1}}. \tag{6}$$

To distinguish symbols of different time, we use one more subscript n ($1 \le n \le N$) to index the N iterations. The symbols, $x_{k,n}$, $y_{k,n}$, $r_{k,n}$, $g_{k,n}$, and $w_{k,n}$, stand respectively for the input, output, sufficient statistic, fading coefficient, and additive noise of the kth subchannel at symbol slot n. Owing to the low duty cycle, successive symbols are far apart in time (see Figure 1). We thus assume different symbols experience independent fading, i.e., the gains $g_{k,n}$ ($1 \le n \le N$) are independent, identically distributed (i.i.d.).

The decision variables obtained at the receiver are the averages of $r_{k,n}$ over the N iterations, denoted as S_k,

$$S_k = \frac{1}{N} \sum_{n=1}^{N} r_{k,n}. \tag{7}$$

By substitution, we rewrite S_k as

$$S_k = \begin{cases} \frac{1}{N}\sum_{n=1}^{N}\left|g_{k,n}\sqrt{\frac{PT_s'}{Q\theta N_0}}+w_{k,n}\right|^2, & k \in \mathcal{S}_m, \\ \frac{1}{N}\sum_{n=1}^{N}\left|w_{k,n}\right|^2, & \text{otherwise,} \end{cases} \tag{8}$$

which in either case are χ^2 random variables with $2N$ degrees of freedom.

Since the average received power of the non-zero tones and the zero tones are $\frac{PT_s'}{Q\theta N_0}+1$ and 1 respectively, the threshold A is given by

$$A = 1 + (1-\epsilon)\frac{PT_s'}{Q\theta N_0}, \tag{9}$$

where ϵ is chosen over $(0,1)$. If there are exact Q of $\{S_k\}$ exceed the threshold, the corresponding $\mathcal{S}_m$ is decoded; otherwise, the receiver declares an error.

Since the scheme transmits $\ln\binom{M}{Q}$ nats of information in NT_s/θ seconds, the data rate R is given by

$$R = \frac{\theta}{NT_s}\ln\binom{M}{Q}. \tag{10}$$

The value of Q is chosen from the integers satisfying $Q \le M/2$, because, for any integer Q greater than $M/2$, we can find an integer $M-Q$ which keeps the number of total symbols the same and leads to a higher peak power per tone. In MFSK, we pick the value of Q to be much smaller than $M/2$.

III. Performance analysis

As we know, an error occurs if $S_k \ge A$ for $k \notin \mathcal{S}_m$ (called *type I error*), or $S_k < A$ for $k \in \mathcal{S}_m$ (called *type II error*). Let E_k be the event that an error occurs at the kth tone. For notational convenience, we define

$$p_{e,1} \triangleq \Pr(E_k) \quad \text{for} \quad k \notin \mathcal{S}_m,$$

and

$$p_{e,2} \triangleq \Pr(E_k) \quad \text{for} \quad k \in \mathcal{S}_m.$$

We will use the same techniques used in [8] and [9] to derive upper bounds and lower bounds of $p_{e,1}$ and $p_{e,2}$, denoted as $p_{e,1}^{(u)}$, $p_{e,2}^{(u)}$, $p_{e,1}^{(l)}$, and $p_{e,2}^{(l)}$.

1) Derivation of $p_{e,1}^{(u)}$: To get the upper bound $p_{e,1}^{(u)}$, we use Chernoff bound

$$\begin{aligned} p_{e,1} &= \Pr\left(NS_k \geq NA\right) \\ &\leq \inf_s E\left[e^{s(NS_k - NA)}\right] \quad \text{(Chernoff bound)} \\ &= e^{-N\Phi(A)}, \end{aligned} \tag{11}$$

where $s \in [0,1]$ and $\Phi\left(A\right)$ is given by

$$\Phi(A) = \sup_s \left[sA - \frac{1}{N}\ln\left(E\left[e^{sNS_k}\right]\right)\right]. \tag{12}$$

Further simplifying $\Phi(A)$, we obtain

$$\begin{aligned} \Phi(A) &= \sup_s \left[sA - \frac{1}{N}\ln\left(E\left[e^{s\sum_{n=1}^{N}|w_{k,n}|^2}\right]\right)\right] && (13) \\ &= \sup_s \left[sA + \ln\left(1 - s\right)\right] && (14) \\ &= A - 1 - \ln A, && (15) \end{aligned}$$

where (13) is obtained by substituting the value of S_k into (12); (14) is based on the fact that $|w_{k,n}|^2$ are independent, exponentially distributed, and the moment generating function of $\sum_{n=1}^{N}|w_{k,n}|^2$ is $(1-s)^{-N}$; (15) is derived by picking $s = (A-1)\diagup A$. We have

$$p_{e,1}^{(u)} = \exp\left[-N\left(A - 1 - \ln A\right)\right]. \tag{16}$$

Note that, $A - 1 - \ln A > 0$ because $A > 1$. The upper bound $p_{e,1}^{(u)}$ decreases to zero as N grows.

2) Derivation of $p_{e,2}^{(u)}$: We use again Chernoff bound to get $p_{e,2}^{(u)}$. We have

$$\begin{aligned} p_{e,2} &= \Pr\left(NS_k \leq NA\right) \\ &\leq \inf_{s<0} E\left[e^{s(NS_k - NA)}\right] \quad \text{(Chernoff bound),} \\ &= e^{-N\Phi(A')}, \end{aligned} \tag{17}$$

where

$$A' = \frac{A}{1 + \frac{PT'_s}{Q\theta N_0}}. \tag{18}$$

The derivation of (17) is as follows,

$$\begin{aligned}
&\inf_{s<0} E\left[e^{s(NS_k-NA)}\right] \\
=\ &\inf_{s<0} \exp\left[-sNA+\ln\left(E\left[e^{s\sum_{n=1}^{N} r_{k,n}}\right]\right)\right] &(19)\\
=\ &\inf_{s<0} \exp\left(-N\left[sA+\ln\left(1-\left[1+\frac{P'T_s}{Q\theta N_0}\right]s\right)\right]\right) &(20)\\
=\ &\exp\left[-N\left(A'-1-\ln A'\right)\right], &(21)
\end{aligned}$$

where (19) is obtained by substituting the value of S_k and rearranging; (20) is due to the fact that $r_{k,n}$ are independent, exponentially distributed, and $E\left[\exp\left(s\sum_{n=1}^{N} r_{k,n}\right)\right]$ is the moment generating function of $\sum_{n=1}^{N} r_{k,n}$; (21) is the result of optimizing (20) over s. We have

$$p_{e,2}^{(u)} = \exp\left[-N\left(A'-1-\ln A'\right)\right]. \tag{22}$$

Since $A' \in (0,1)$ and $A'-1-\ln A' > 0$, the upper bound $p_{e,2}^{(u)}$ decreases to zero as $N \to \infty$.

3) Derivation of $p_{e,1}^{(l)}$*:* Since S_k are χ^2 random variables with $2N$ degrees of freedom, we can use the result in [13, §2.1.4] to evaluate the cumulative distribution functions. The probability $p_{e,1}$ is

$$\begin{aligned}
p_{e,1} &= \Pr\left(\sum_{n=1}^{N} |w_{k,n}|^2 > NA\right)\\
&= \exp\left(-NA\right)\sum_{k=0}^{N-1}\frac{(NA)^k}{k!}, &(23)
\end{aligned}$$

by directly applying the formula in [13, §2.1.4]. Since $(NA)^k / k!$ is positive for all k, we have

$$\sum_{k=0}^{N-1}\frac{(NA)^k}{k!} \geq \frac{(NA)^{N-1}}{(N-1)!}, \tag{24}$$

which leads to

$$p_{e,1} \geq \exp\left[-NA+\ln\left(\frac{(NA)^{N-1}}{(N-1)!}\right)\right] \tag{25}$$

by using (24) in (23). Applying Stirling's approximation,

$$\sqrt{2\pi N}N^N e^{\left(-N+\frac{1}{12N+1}\right)} < N! < \sqrt{2\pi N}N^N e^{\left(-N+\frac{1}{12N}\right)},$$

we obtain

$$p_{e,1}^{(l)} = \exp\left(-N\left[A-1-\ln A+o_1(N)\right]\right), \tag{26}$$

where $o_1(N)$ is a vanishing term as N increases, given by

$$o_1(N) = \frac{1}{2N}\ln\left(2\pi N A^2\right) + \frac{1}{12N^2}. \tag{27}$$

The lower bound $p_{e,1}^{(l)}$ decreases to zero as $N \to \infty$.

4) Derivation of $p_{e,2}^{(l)}$: For the probability $p_{e,2}$, we have

$$\begin{aligned} p_{e,2} &= Pr\left(\sum_{n=1}^{N}\left|g_{k,n}\sqrt{\frac{P'T_s}{Q\theta N_0}} + w_{k,n}\right|^2 < NA\right) \\ &= \exp\left(-NA'\right)\sum_{k=N}^{\infty}\frac{\left(NA'\right)^k}{k!}. \end{aligned} \tag{28}$$

Since the inequality

$$\sum_{k=N}^{\infty}\frac{\left(NA'\right)^k}{k!} \geq \frac{\left(NA'\right)^N}{N!} \tag{29}$$

holds as $\left(NA'\right)^k / k!$ is positive for all k, we can bound $p_{e,2}$ by applying (29) to (28),

$$p_{e,2} \geq \exp\left(-NA' + \ln\left[\frac{\left(NA'\right)^N}{N!}\right]\right). \tag{30}$$

Using Stirling's approximation, we obtain

$$p_{e,2}^{(l)} = \exp\left[-N\left(A' - 1 - \ln A' + o_2(N)\right)\right], \tag{31}$$

where

$$o_2(N) = \frac{1}{2N}\ln(2\pi N) + \frac{1}{12N^2} \tag{32}$$

is a diminishing term as N increases. The lower bound $p_{e,2}^{(l)}$ decreases to zero as $N \to \infty$.

Observe that, the difference between $p_{e,1}^{(u)}$ and $p_{e,1}^{(l)}$ is a vanishing term in the coefficient of N. Therefore, these two bounds decrease in the same order as $N \to \infty$. For the same reason, $p_{e,2}^{(u)}$ and $p_{e,2}^{(l)}$ also decrease in the same order as $N \to \infty$.

We represent $p_{e,1}^{(u)}$, $p_{e,2}^{(u)}$, $p_{e,1}^{(l)}$, and $p_{e,2}^{(l)}$ as functions of N. In fact, for any given data rate, N is directly related to M, the number of total tones, according to (10). The value of N is monotonically increasing with M. In the following discussion, we will frequently substitute N with M by using (10) to show the effect of M on the scheme's performance. In particular, we are interested in writing the probability of error as a function of M. This way, we can characterize how the infinite bandwidth limit is approached as M tends to infinity, and hence obtain the insights on how to design the optimal signaling at a finite but large bandwidth.

A. Bounds on the error probability

1) Upper bound on the error probability: Using the union bound, the error probability is upper bounded by

$$p_e \leq (M-Q)\, p_{e,1} + Q p_{e,2}. \tag{33}$$

Since all symbols have the identical union bound, we need not to average the bound all over the symbol set.

Noticing that the inequality

$$M - Q \leq Q \binom{M}{Q}^{1/Q}$$

is always true for $M > Q$, we have

$$\begin{aligned} (M-Q)\, p_{e,1} &\leq Q \binom{M}{Q}^{1/Q} p_{e,1} \\ &\leq Q \binom{M}{Q}^{1/Q} p_{e,1}^{(u)}. \end{aligned} \tag{34}$$

Denote the upper bound of $(M-Q)\, p_{e,1}$ in (34) as $p_{e,\bar{\mathcal{S}}_m}^{(u)}$. By substituting the expressions of $p_{e,1}^{(u)}$, A, and N into (34), we obtain

$$\begin{aligned} p_{e,\bar{\mathcal{S}}_m}^{(u)} &= Q \exp\left(-\ln \binom{M}{Q} \left[\frac{(1-\epsilon)\, PT_s'}{QN_0RT_s} - \frac{1}{Q}\right.\right. \\ &\qquad \left.\left. - \frac{\theta}{RT_s} \ln\left(1 + \frac{(1-\epsilon)\, PT_s'}{Q\theta N_0}\right)\right]\right). \end{aligned}$$

On other hand, an upper bound of $Qp_{e,2}$ is given by $Qp_{e,2}^{(u)}$, denoted as $p_{e,\mathcal{S}_m}^{(u)}$. Substituting A' and N into $p_{e,2}^{(u)}$, we have the expression of $p_{e,\mathcal{S}_m}^{(u)}$,

$$\begin{aligned} p_{e,\mathcal{S}_m}^{(u)} &= Q \exp\left(-\frac{\theta \ln \binom{M}{Q}}{RT_s} \left[\frac{-\epsilon PT_s'}{Q\theta N_0 + PT_s'}\right.\right. \\ &\qquad \left.\left. - \ln\left(1 - \frac{\epsilon PT_s'}{Q\theta N_0 + PT_s'}\right)\right]\right). \end{aligned}$$

Such, we can write an upper bound of the symbol error probability as

$$p_e \leq p_{e,\bar{\mathcal{S}}_m}^{(u)} + p_{e,\mathcal{S}_m}^{(u)}. \tag{35}$$

This upper bound holds for any $\epsilon \in (0,1)$. Since our goal is to derive an upper bound, picking any specific ϵ will serve this purpose. Instead of the optimal value of ϵ minimizing the sum, we pick a value simplifying our analysis. Yet, as we can see later, the corresponding upper bound is tight in the wideband regime.

Notice that $p^{(u)}_{e,\overline{\mathcal{S}}_m}$ strictly increases with ϵ, while $p^{(u)}_{e,\mathcal{S}_m}$, on the contrary, strictly decreases. We choose the value of ϵ letting these two bounds equal, which is

$$\begin{aligned}\epsilon^* &= \frac{Q\theta N_0 + PT'_s}{PT'_s}\left[1 - \frac{RN_0T_s}{PT'_s}\right.\\&\left.-\frac{Q\theta N_0}{PT'_s}\ln\left(1+\frac{PT'_s}{Q\theta N_0}\right)\right].\end{aligned} \tag{36}$$

To keep $\epsilon^* \in (0,1)$, the data rate R must in the range

$$0 \le R < \frac{PT'_s}{N_0T_s} - \frac{Q\theta}{T_s}\ln\left(1+\frac{PT'_s}{Q\theta N_0}\right). \tag{37}$$

With (37) holds, by substituting ϵ^* into $p^{(u)}_{e,\overline{\mathcal{S}}_m} + p^{(u)}_{e,\mathcal{S}_m}$, we obtain an upper bound of the error probability

$$p_e \le 2Q\exp\left(-\ln\binom{M}{Q}E_r(M,Q,T_s,R,\theta)\right), \tag{38}$$

where

$$E_r(M,Q,T_s,R,\theta) = \frac{\theta}{RT_s}\left[\beta - 1 - \ln\beta\right], \tag{39}$$

and

$$\beta = \frac{RN_0T_s}{PT'_s} + \frac{Q\theta N_0}{PT'_s}\ln\left(1+\frac{PT'_s}{Q\theta N_0}\right). \tag{40}$$

By examining (40), we conclude that $\beta \ge 0$ for any feasible choice of (M,Q,T_s,R,θ). On the other hand, because the data rate should satisfy $R < \frac{PT'_s}{N_0T_s}$ for reliable communications according to (37), and the duty cycle θ decreases to zero with increasing bandwidth as we suggest, we have $\beta < 1$ in the wideband regime.

2) Lower bound on the error probability: The error probability could be lower bounded by

$$\begin{aligned}p_e &\ge \sum_{k=1}^{M}\Pr(E_k) - \sum_{k\neq j}\Pr(E_k\cap E_j)\\&\ge (M-Q)\,p^{(l)}_{e,1} + Qp^{(l)}_{e,2} - (M-Q)\,Qp^{(u)}_{e,1}p^{(u)}_{e,2}\\&\quad-\binom{M-Q}{2}p^{(u)^2}_{e,1} - \binom{Q}{2}p^{(u)^2}_{e,2}.\end{aligned} \tag{41}$$

Recall that, $p^{(u)}_{e,1}$, $p^{(u)}_{e,2}$, $p^{(l)}_{e,1}$, and $p^{(l)}_{e,2}$ are all decreasing functions of N. $(M-Q)\,p^{(l)}_{e,1}$ and $(M-Q)\,p^{(u)}_{e,1}$ decrease with N in the same order, and so do $Qp^{(l)}_{e,2}$ and $Qp^{(u)}_{e,2}$. If $(M-Q)\,p^{(l)}_{e,1}$

decrease faster than $Qp_{e,2}^{(l)}$ in N, then the lower bound (41) is of the order of $Qp_{e,2}^{(l)}$, and all other four terms are dominated. Otherwise, the lower bound has the order of $(M-Q)\,p_{e,1}^{(l)}$. That is,

$$p_e \geq \max\left((M-Q)\,p_{e,1}^{(l)}, Qp_{e,2}^{(l)}\right). \tag{42}$$

As N approaches infinity, the coefficient of N in the exponent of $(M-Q)\,p_{e,1}^{(l)}$ is given by

$$\begin{aligned}
& \lim_{N\to\infty} \frac{\ln\left[(M-Q)\,p_{e,1}^{(l)}\right]}{N} \\
= & \lim_{N\to\infty} \left[\frac{\ln p_{e,1}^{(l)}}{N} + \frac{\ln\binom{M}{Q}}{QN}\right] & (43)\\
= & \lim_{N\to\infty} \frac{\ln p_{e,1}^{(l)}}{N} + \frac{RT_s}{Q\theta} & (44)\\
= & -A + 1 + \ln A + \frac{RT_s}{Q\theta}. & (45)
\end{aligned}$$

To obtain (43), we used

$$\lim_{N\to\infty} \frac{\ln(M-Q)}{N} = \lim_{N\to\infty} \frac{\ln\binom{M}{Q}}{QN},$$

where Q is predetermined, and, for a given data rate, M increases with N according to (10).

As N approaches infinity, the coefficient of N in the exponent of $Qp_{e,2}^{(l)}$ is given by

$$\begin{aligned}
& \lim_{N\to\infty} \frac{\ln\left(Qp_{e,2}^{(l)}\right)}{N} \\
= & \lim_{N\to\infty} \frac{\ln p_{e,2}^{(l)}}{N} \\
= & -A' + 1 + \ln A'. & (46)
\end{aligned}$$

The values of (45) and (46) are both negative. The value of (45) strictly increases with ϵ. Therefore, the larger ϵ is, the slower $(M-Q)\,p_{e,1}^{(l)}$ decreases with N. The value of (46), on the contrary, strictly decreases with ϵ. The larger ϵ is, the faster $Qp_{e,2}^{(l)}$ decreases with N. Thus, the lower bound (42) depends on the value of ϵ picked.

If we pick the value ϵ^* given by (36), then the values of (45) and (46) are equal, which let $(M-Q)\,p_{e,1}^{(l)}$ and $Qp_{e,2}^{(l)}$ decrease with N in the same order. Other values of ϵ will let one of these two terms decrease more slowly. Since (42) is determined by the larger term of $(M-Q)\,p_{e,1}^{(l)}$

and $Qp_{e,2}^{(l)}$, the value of ϵ other than ϵ^* will lift the lower bound upwards. Consequently, by picking ϵ^*, we get a lower bound for p_e in the wideband regime for all $\epsilon \in (0,1)$,

$$p_e \geq Q \exp\left(-\ln\binom{M}{Q} E_r(M,Q,T_s,R,\theta)\right), \tag{47}$$

where $E_r(M,Q,T_s,R,\theta)$ was given by (39).

B. The capacity-achieving property

In [2], Telatar and Tse have proven that, in multipath fading channels, single-tone FSK can achieve the capacity of an AWGN channel with the same average received power at the wideband limit. We now prove that MFSK has the same capacity-achieving property in the wideband limit.

Theorem 1: MFSK can achieve any data rates that satisfy

$$R < \left(1 - \frac{\mathcal{L}}{T_s}\right) \frac{P}{N_0} \tag{48}$$

with an arbitrarily small probability of error over a multipath fading channel with average power constraint P, by using bandwidth large enough.

Proof: Since the inequality $\beta - 1 > \ln\beta$ holds for all $\beta \in [0,1)$, we have $E_r(M,Q,T_s,R,\theta) > 0$ in (39). As a result, the upper bound (38) decreases to zero when M increases to infinity, as long as (37) holds. That is, for the data rates in the range defined by (37), we can use MFSK to transmit data with arbitrarily small error probability by using bandwidth large enough.

The data rate R can get arbitrarily close to $\frac{PT_s'}{N_0 T_s}$, i.e., $\left(1 - \frac{\mathcal{L}}{T_s}\right)\frac{P}{N_0}$, by letting θ decrease to zero as the bandwidth increases to infinity. Hence, as long as the data rate satisfies (48), the MFSK scheme can achieve arbitrarily small error probability by picking bandwidth large enough.

■

Recalling that the channel is assumed to be underspread and $\mathcal{L} \ll T_s$, we conclude that the capacity of MFSK approaches Shannon's infinite bandwidth capacity limit $\frac{P}{N_0}$.

In the proof, we let θ decrease to zero as the bandwidth increases. However, θ must decrease more slowly than $1 \big/ \ln\binom{M}{Q}$ as M increases. Otherwise, by substituting (39) into (38), we can verify that (38) will not decrease with M. Intuitively, it is because decreasing θ will reduce the information rate. To counteract this effect, we need to increase the information bits per symbol, i.e., increase $\ln\binom{M}{Q}$. If θ decreases too fast with M, the achievable data rate will be compromised.

At a particular finite but large bandwidth, the capacity of the MFSK schemes with different Q may vary. In such cases, considering the whole family of the MFSK schemes, including single-tone FSK, has the benefit of more flexibility in the tradeoff between the spectrum efficiency and the energy efficiency.

C. The error exponent for MFSK

In [8] and [9], Lun et al. discussed the error exponent for single-tone FSK. We will discuss the error exponent of MFSK in this subsection.

The merit of the study of the error exponents for these schemes is, as will become clear later on, a characterization of how the error performance is affected by the channel parameters including the SNR, rate, duty cycle, bandwidth, and Q.

Theorem 2: As bandwidth increases to infinity, MFSK has the following relation between the error probability p_e and the total number of symbol $\binom{M}{Q}$:

$$\lim_{M\to\infty} \frac{-\ln p_e}{\ln \binom{M}{Q}} = E_r(M, Q, T_s, R, \theta), \tag{49}$$

where $E_r(M, Q, T_s, R, \theta)$ is defined in (39).

Proof: According to (47), we have

$$\lim_{M\to\infty} \frac{-\ln p_e}{\ln \binom{M}{Q}} \leq E_r(M, Q, T_s, R, \theta). \tag{50}$$

The reverse inequality follows from (38),

$$\lim_{M\to\infty} \frac{-\ln p_e}{\ln \binom{M}{Q}} \geq E_r(M, Q, T_s, R, \theta). \tag{51}$$

Combining the inequalities (50) and (51), we obtain (49), thus completing the proof. ∎

Therefore, $E_r(M, Q, T_s, R, \theta)$ represents the true exponential dependence of the error probability on $\ln \binom{M}{Q}$ for M sufficiently large. We call $E_r(M, Q, T_s, R, \theta)$ the error exponent of MFSK which is analogous to the treatment of random block coding over discrete memoryless channels by Gallager [6, §5.8].

The error probability of Q-tone FSK decreases with M as $\binom{M}{Q}^{-E_r} \approx M^{-QE_r}$, whereas the error probability of single-tone FSK decays as M^{-E_r}. In the case that the system has a peak power constraint, i.e., a constraint on $\frac{P}{Q\theta}$, the error exponents for single-tone FSK and Q-tone FSK, at a given data rate, are approximately equal. Thus the error probability of the Q-tone FSK scheme decays with M faster, as shown in Figure 2.

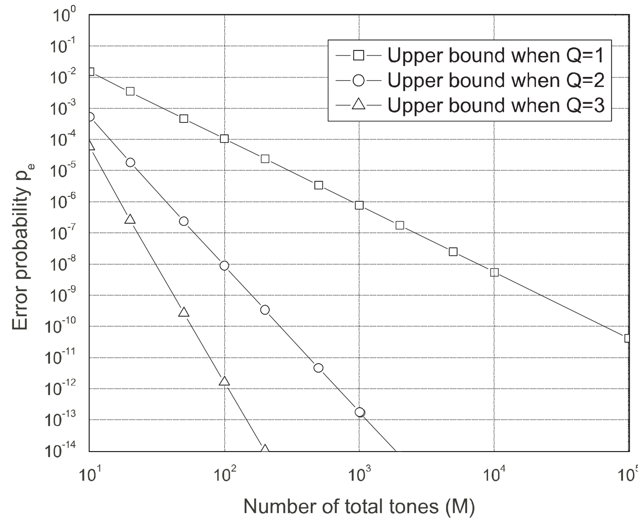

Fig. 2. Upper bounds of error probability for single-tone FSK, two-tone FSK, and three-tone FSK when peak power, duty cycle, and data rate of these three schemes are the same.

IV. Numerical results

In the previous sections, we have studied achievable rates and error probabilities of MFSK. We now proceed to evaluate the capacity of MFSK schemes with a simple hard-decision receiver for particular parameter choices. We show that, even with finite bandwidth, our scheme yields achievable rates that are for practical purposes very close to the wideband capacity limit.

Recall that, there are $\binom{M}{Q}$ symbols in a Q-tone FSK scheme. We can use an equivalent discrete memoryless channel (DMC) with $\binom{M}{Q}$ inputs and $\binom{M}{Q}$ outputs for capacity calculation. Apparently, due to the symmetry of the DMC, the capacity-achieving input for the DMC is uniformly distributed. If we can obtain the transition probabilities between the input and the output, we can directly calculate the capacity. The transition probabilities depend on the detection rule, and are, for most of the cases, intractable. We restrict ourselves to the single-tone case and the two-tone case using a symbol-by-symbol detection scheme based on the maximum-a-posteriori (MAP) rule.

The symbol-by-symbol decision is optimal from a symbol detection point of view, since the channel is memoryless, but may not be optimal from a codeword decoding point of view. In effect, we are considering a simple hard-decision decoder instead of a soft-decision decoder. Hard-decision decoders have information loss during the symbol-by-symbol detection, while soft-decision decoders use these information to decode symbols jointly. Our results show that, even with a simple hard-decision receiver, MFSK yields achievable rates very close to the wideband capacity limit.

As we know, the capacity C of a fading channel is bounded by the AWGN capacity,

$$C \leq W \ln\left(1 + \frac{P}{N_0 W}\right), \tag{52}$$

where W is the bandwidth of the system. We call this bound the *power-limited bound*, because the capacity of MFSK is tightly bounded by this bound in the power-limited regime.

Another upper bound is derived from the DMC model,

$$C \leq \frac{1}{T_s} \ln \binom{M}{Q}, \tag{53}$$

because the entropy per input symbol is $\ln \binom{M}{Q}$. This bound is tight when the system has a "less than enough" bandwidth. We denote this bound as the *bandwidth-limited bound*.

First, we consider a single-tone FSK scheme and a two-tone FSK scheme with the same bandwidth constraint. We compare their performance in terms of capacity versus signal-to-noise ratio. We assume the symbol time, $T_s = 10\mu s$, the multipath spread, $\mathcal{L} = 1\mu s$, and the system bandwidth, $W = 1MHz$. The SNR ($\frac{P}{N_0 W}$) ranges from 10^{-2} to 10^3. We plot the capacity curves and the upper bounds in Figure 3.

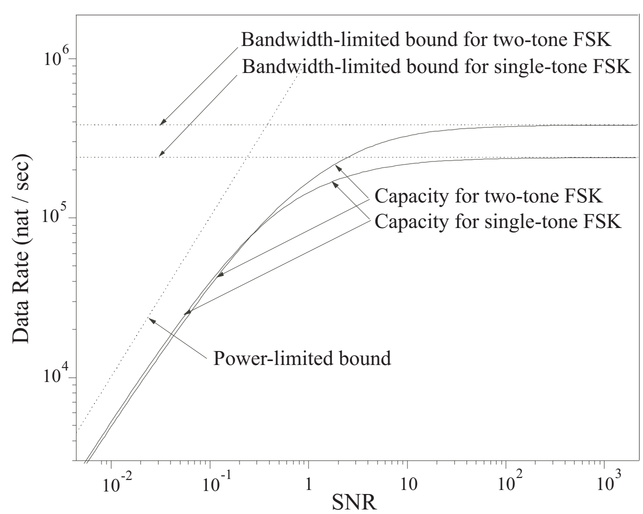

Fig. 3. The capacity of single-tone FSK and two-tone FSK for $T_s = 10\mu s$, $\mathcal{L} = 1\mu s$, and $W = 1MHz$.

We can make the following observations from Figure 3:

- The capacity of the single-tone FSK and of the two-tone FSK are very close to the upper bounds both in the power-limited regime (the SNR ranges from 10^{-2} to 1) and the bandwidth-limited regime (the SNR ranges from 1 to 10^3).
- In the power-limited regime, the performances for these two schemes are very close, and the power-limited bound is tight in this regime, which is roughly 2 dB higher than the capacities. Comparing to the capacity of CDMA [5], OFDM [14], and PPM [15] in the wideband regime, the capacity of MFSK is very close to the AWGN capacity.
- In the bandwidth-limited regime, the performance of two-tone FSK is better than that of single-tone FSK because of its larger input set (higher spectrum efficiency).

In Figure 4, we assume the average power is fixed, where $T_s = 0.1s$, $\mathcal{L} = 1\mu s$, and $\frac{P}{N_0} = 40Hz$. The bandwidth ranges from $100Hz$ to $10KHz$. We observe that, in the bandwidth-limited regime (from $100Hz$ to $1KHz$), the performance of two-tone FSK is better than that of single-tone FSK. In the regime that the bandwidth is smaller than $100Hz$, however, two-tone FSK shows no advantage at all. This is because that the number of total frequencies, M, is so small that $\binom{M}{1}$ and $\binom{M}{2}$ are comparable.

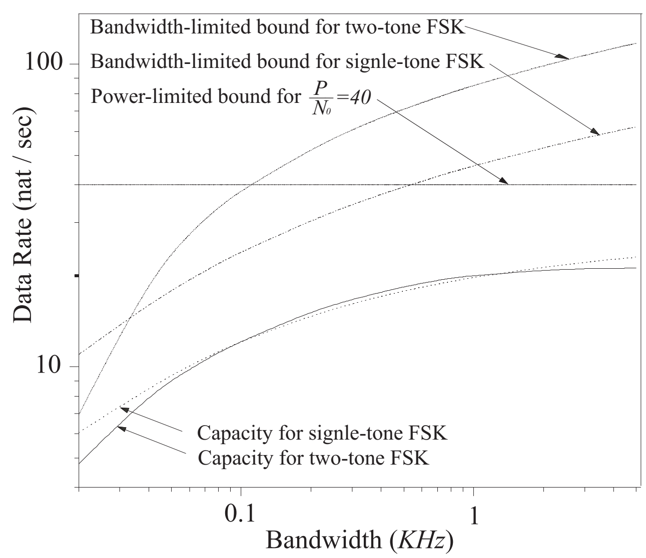

Fig. 4. Capacity versus bandwidth curves for $T_s = 0.1s$, $\mathcal{L} = 1\mu s$, and $\frac{P}{N_0} = 40$.

Our examples illustrate that, even for finite bandwidth, multi-tone FSK schemes can yield rates that are appreciably close to the wideband capacity limit. The ability to vary the number of tones according to the transmission condition (average power, bandwidth constraint) provides a useful design parameter which can be adapted to different operating regimes.

V. Conclusions

In this paper, we introduced a class of multi-tone FSK schemes for non-coherent communications in wideband Rayleigh fading channels. For a Rayleigh fading channel with infinite bandwidth, the AWGN capacity limit can be achieved by using multi-tone FSK schemes.

For finite bandwidth applications, we derived an upper bound and a lower bound for the symbol error probability. These two bounds lead to the error exponent of multi-tone FSK, which describes how the error probability depends on the different parameters. According to our results, the multi-tone FSK has comparable error performance as the single-tone FSK.

We also calculated the achievable rates of single- and two-tone FSK schemes with finite bandwidth. Our result shows that we can attain achievable rates very close to capacity in fading channels.

Considering the low duty cycle of MFSK, we can easily extend our scheme to multiple-access cases by orthogonalizing different users' channels in time. Another direction for future research could be considering soft decoding/partially soft decoding in the MFSK scheme. By tracking all/several multiple-symbol codewords over a certain period of time, we can decode the codeword with a system that approximates better than our hard-decision scheme, a maximum-likelihood decoder. By doing so, we could further close the gap between the achievable data rates and the capacity limit.

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List of Figure Captions

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