2 DIGITAL MODULATION

Why Digital Modulation

The RF spectrum is limited and has to be divided among an increasing numbers of services and users. With digital modulation schemes and their effects on overall system efficiency it is possible to obtain greater capacity and convey more information than with analogue ones in the same bandwidth.

Developers of radio communication systems face the following constraints:

- Available bandwidth
- Permissible power
- Noise level of the system

Different types of digital modulation exists, each with its pros and cons. Examples are:

- ASK (Amplitude Shift Keying)
- FSK (Frequency Shift Keying)
- PSK (Phase Shift Keying)
- QPSK (Quadrature Phase Shift Keying)
- MSK (Minimum Shift Keying)
- QAM (Quadrature Amplitude Modulation)
- $\pi/4$ DQPSK (Differential Quadrature Phase Shift Keying)

Initially, in digital transmission the information to be transmitted is in the form of a sequence a(n) comprising of elements from a set $a, a \in \{0,1\}$. The modulator converts the sequence of logical ones and zeros into a sequence of analogue signals suitable for transmission in the RF channel. The conversion of the digital signals into baseband signals is referred to as baseband modulation while the conversion into RF signals is referred to as RF carrier modulation.

This section gives an introduction to the main types of the digital modulation methods.

Noise in the Communication Channel

In a communication channel it will be noise and other distortions that degrade the capacity.

Fundamental Data Capacity

Shannon showed in his famous paper of the fundamentals of data communication that the capacity of the Gaussian white noise limited communication channel is:

$$C = B \cdot \log_2(1 + \frac{S}{N})$$

where C is channel capacity in bit/second, B is the bandwidth of the channel in Hz, S is the signal power in Watts and N is the noise power in Watts.

Shannon's formula is a theoretical limit and practical implementations are worse than the capacity bound given by the formula.

Signal to Noise Ratio, E_b/N₀

The noise most commonly used in compering digital modulation methods is the *Additive White Gaussian Noise* (AWGN). The white noise is the noise you hear in an FM broadcast receiver (without muting) when tuned off stations, and it is called white due to it contains frequency components from low frequency into the microwave region. In a receiver the noise spectrum will be limited with the transfer function of the receiver, but the noise energy that passes will add to the datasignal and decrease the distance between signalling states.

In digital systems a common terminology for describing the signal-to-noise ratio is the E_b/N_0 , the energy of a single data bit to the noise power density. E_b/N_0 is a dimensionless quantity and is the signal-to-noise ratio per bit. It is often expressed in dB.

The amplitude of the noise voltage is random. As an example at one moment the amplitude is 0.2 Volt and in the next one it is 1.0 Volt. In calculation of the Bit Error Rate (BER) the Gaussian distribution is used for noise voltage amplitude. The use of Gaussian distribution closely models what happens in the real world. To calculate the BER one question has to be answered: How often does the noise voltage exceed a certain signal amplitude of the opposite polarity. This is not easy to answer but a mathematical function, the complementary error function erfc(), has been defined to answer the question. The reader will find more about the error function in math text books. More information of BER calculations and the complementary error function is found in later sections, but first some fundamentals of signalling is explained.

Signalling, Antipodal, Orthogonal and Non-Orthogonal

Digital transmission formats in general fall into three types:

Antipodal The signalling states are exact opposites of one another. Antipodal means

that the signalling states (phase states) are as far apart in the phase plane

as possible.

Orthogonal The signalling states have a phase difference of 90 degrees and do not

affect each other

Non-orthogonal The signalling states affects each other to some degree, usually negatively.

Antipodal Signalling

Two level PSK (2PSK) is antipodal signalling. A phase shift of 180 degrees is used for the two signalling states, ("1") and ("0"). Figure 2-1 shows the phaseshift on a carrier and how it is viewed in the phase plane.

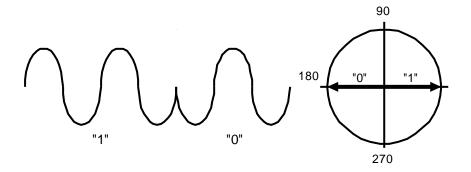


Figure 2-1: Phase states of antipodal signalling

As shown in figure 2-1 the phase shifts 180 degrees when signal to be transmitted shifts from "1" to "0". In the PSK demodulator to detectors can be used, one for the in-phase signal with the carrier and one for the out-of-phase signal with the carrier. The two detectors have output of opposite polarity, and when the phase shifts the polarity of the outputs of the detectors changes. The two different phase detectors are always both participating in the decision about what value the output data is going to be. Each state affects both channels, and this is called antipodal signalling.

Orthogonal Signalling

The definition of orthogonality can be shown mathematically and visualised with multiplication of two waveforms. S_i and S_g are the signals which are cosine and sine ones.

$$s_{i}(t) := cos(\omega t)$$
 $s_{q}(t) := sin(\omega t)$

A frequency of one megahertz is used in the example. The equation below shows the result of the two signals S_i and S_q multiplied and integrated over a time period of one microsecond, the cycle time of the signals.

$$\int_{0}^{(1\cdot10)^{-6}} s_{i}(t) \cdot s_{q}(t) dt = 0$$

The two signals S_i and S_q are orthogonal over time interval T when the equation above is zero, as is the case. Thus cosine and sine waves are orthogonal. This property of the cosine and sine waves are used in IQ modulation which are popular in many applications of digital transmitters.

Bit Error Rate (BER) calculations for PSK and FSK

The following equations and curves in figure 2-2 shows BER calculations for PSK and FSK with coherent and non-coherent demodulation. (Coherent and non-coherent demodulation is not within the scope of this paper).

erfc(x) := 1 - erf((v)	The complex	mentary erro	r function	orfc(x)
CIIC (<i>X)</i> 1 - CII((\mathbf{A})	The complet	nemary erro	i junction	$er_{I}((x))$

$$cPSK(x) := \frac{1}{2} \cdot erfc\left(\sqrt{x}\right)$$
 This formula gives the BER for 2PSK, antipodal signalling. 2PSK is normally used as reference of other digital modulation methods

cFSK(x) :=
$$\frac{1}{2}$$
 · erfc $\left(\sqrt{\frac{x}{2}} \right)$ This formula gives the BER for 2FSK signalling coherently demodulation

NocFSK(x) :=
$$\frac{1}{2} \cdot e^{-\frac{x}{2}}$$
 This formula gives the BER for 2FSK signalling non-coherently demodulation.

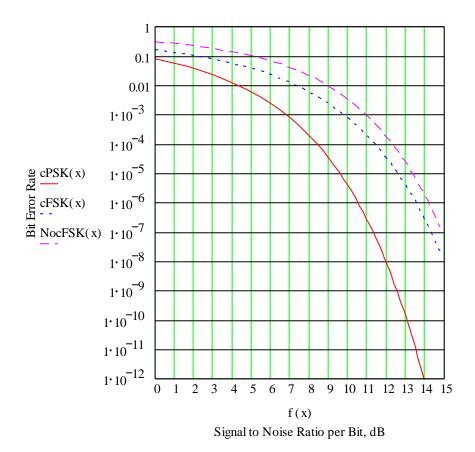


Figure 2-2: Digital modulation performance vs Signal to Noise Ratio

As can be seen form the curves of figure 2-2 a 10 dB of E_b/N_0 gives a BER of about $2 \cdot 10^{-5}$ for PSK and about 10^{-3} for FSK, both coherently demodulated. From the curves it is found that to obtain a given BER FSK needs an additional 3 dB in the signal-to-noise ratio compared to PSK. This is due to the fact that in PSK information from both the signalling states, 0 and 180 degrees, are used to determine symbol received. In FSK only information from one signal is used, it is no opposite frequency of the one used for a symbol transmission. The signal information output from the detector is then twice for PSK compared to FSK which gives the 3 dB improvement.

Non-coherent demodulated FSK has higher requirement to signal-to-noise ratio than coherent demodulated FSK. As can be seen from the curves the penalty paid with using non-coherent demodulation is small, and it is used in many applications due to it's lower implementation complexity.

IQ-Modulation

The GSM specification for maximum frequency and angle error during a transmission burst are so tight that modulating a VCO would be incapable to meet the requirements. An IQ modulator provides better results and is popular in many applications. This section briefly reviews IQ modulation.

IQ Formats

The modulation can be expressed in terms of I, the inphase component, and Q, the quadrature component, of the signal. The I- and Q-components map amplitude and phase in the IQ plane, and the points in the plane are called constellation points. Figure 2-3 shows the IQ plane with three signals at different constellation points.

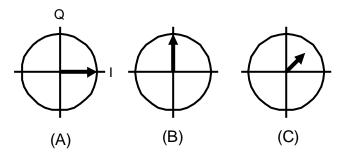


Figure 2-3: IQ plane

In (A) the I-component is unity and the Q-component is zero. The amplitude is unity and phase is zero. In (B) I-component is zero and Q-component is unity. Amplitude is unity but the phase of the signal is +90 degrees. In (C) the I- and Q-components are 0.5. Amplitude is now 0.707 and phase is +45 degrees. Amplitude- and phase modulation is done with changing the I- and Q-components.

In the following example some mathematics of IQ modulation are shown. The signals $s_i(t)$ and $s_q(t)$ are the I- and the Q-components which are multiplied with a modulation signal with amplitude A. Amplitude values used are: $A_1 = 1$, $A_2 = 0$, $A_3 = 0$, $A_4 = 1$, $A_1 = \sqrt{2}/2$ and $A_1 = \sqrt{2}/2$. The following equations, $m_1(t)$, $m_2(t)$, $m_3(t)$, show how the modulation is applied. Figure 2-4 shows the curves of the signals $m_1(t)$, $m_2(t)$, $m_3(t)$.

$$m_1(t) := A_1 \cdot s_i(t) + A_2 \cdot s_q(t)$$

 $m_2(t) := A_3 \cdot s_i(t) + A_4 \cdot s_q(t)$
 $m_3(t) := A_5 \cdot s_i(t) + A_6 \cdot s_q(t)$

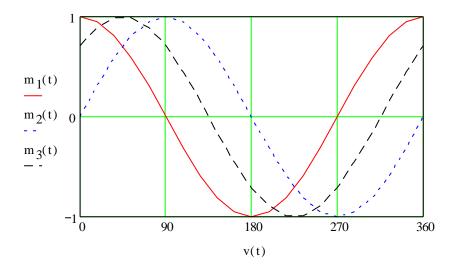


Figure 2-4: I/Q modulation

In $m_1(t)$ the I-signal is unity and Q-signal is zero. In $m_2(t)$ the I-signal is zero and the Q-signal is unity. The curves in figure 2-4 shows $m_2(t)$ 90 degrees in advance of $m_1(t)$ as expected. In $m_3(t)$ both the I-and Q-signals are square root of two half. The phase difference is now 45 degrees compared to $m_1(t)$. Figure 2-5 shows the IQ modulator implementation.

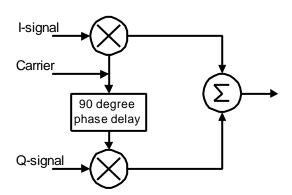


Figure 2-5: IQ modulator implementation

With the IQ-modulator precise control of the amplitude and phase of the carrier is obtained.

Digital Modulation Formats

This section explains the main digital modulation formats, their spectral efficiency and properties for practical systems.

Modulation Types

The baseband digital data signal is to be modulated onto a carrier for transmission, and the modulation can be carried out in different ways with different properties. Important characteristics of the modulated signal is:

- Emitted signal bandwidth
- Error performance
- Implementation complexity

Three distinct properties of the carrier can be changed: Amplitude, frequency and phase. In analogue modulation it is known as AM (Amplitude Modulation), FM (Frequency Modulation) and PM (Phase Modulation). Their digital representations are ASK (Amplitude Shift keying), FSK (Frequency Shift Keying) and PSK (Phase Shift Keying).

A comparison of the digital modulation methods for practical applications is found in Appendix D.

Amplitude Shift Keying (ASK)

In amplitude shift keying the amplitude of the carrier is varied following the binary source (digital baseband signal). In the simplest form the carrier is turned on and off to represent the ones and zeroes. This is known as on-off keying (OOK). An example is Morse code transmission where the alphabet is a combination of short and long pulses of the carrier. Figure 2-6 shows an example of the amplitude modulated carrier using OOK.

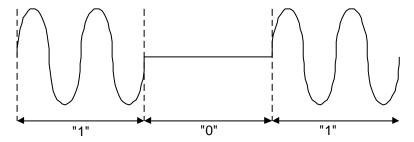


Figure 2-6: Amplitude shift keying

One problem with the OOK is the interference and noise in the receiver in time intervals of no transmission.

A carrier with no modulation needs an infinite small bandwidth to be received. Keying the carrier on and off produces sidebands (upper and lower) corresponding to the period of the keying. A keying frequency of 50 Hz will have sidebands of multiples of 50 Hz above and below the carrier. The distribution of power in the sidebands depends on the rise- and fall-time of the pulsed carrier. Using a receiver with narrow bandwidth the noise power will be small and the narrowband modulation in case of

Morse code is possible to detect in channels with high level of noise and interference as often found in the HF-band (3 MHz to 30 MHz).

ASK is not used in many applications. Amateur Morse code transmission is one example of current usage.

Frequency Shift Keying (FSK)

FSK is a simple form of modulation and has been used for many years due to it's low implementation complexity. FSK is constant envelope modulation, that is the amplitude of the RF-signal is constant. This simplifies transmitter amplifier which can be non-linear, class C, with high efficiency. Examples of modern applications using FSK are paging systems and cordless telephone systems including DECT (Digital Enhanced Cordless Telephone).

In FSK one frequency is used to represent the logical "1" and another frequency the logical "0". This modulation is called 2FSK. Two level FSK can be viewed as a special form of amplitude modulation with to different carriers switched on and off as shown in figure 2-7.

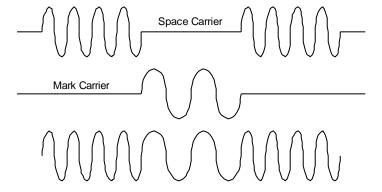


Figure 2-7: FSK viewed as a sum of two amplitude modulated carriers.

Random digital data has a $\sin(x)/x$ rolloff. To obtain optimal demodulation the carriers shall be spaced so that spectral nulls from current carrier is at the other carrier frequency. Figure 2-8 shows a system with $\pm 1/2T$ deviation.

The FSK spectrum is a superposition of two spectra. As shown in figure 2-8 when one carrier is at maximum the other one is at zero. The signalling states are orthogonal when it is no cross-talk between them as with the $\pm 1/2$ T deviation. The real part of the orthogonality is zero when the carriers are spaced 1/2T apart which is equal to $\pm 1/4$ T deviation. This is known as Minimum Shift Keying.

FSK modulation can be done in different ways. In case of a transmitter with a VCO (Voltage Controlled Oscillator) as carrier source it can be modulated with shifting the frequency with the signalling state to be transmitted. One problem arise when the baseband signal has a spectrum from DC. The VCO controlling loop will try to compensate for frequency changes that fall within the loop passband, as

will be the case with the low frequency content of the baseband modulating signal. One solution to this problem is to modulate the reference oscillator with the low frequency content and the VCO with the high frequency content. Another method to do the modulation is to use IQ-modulation.

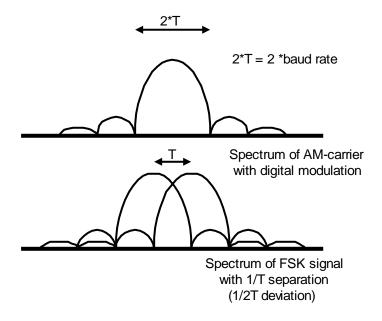


Figure 2-8: FSK modulation with $\pm 1/2T$ deviation

Phase Shift Keying (PSK)

In PSK the phase of the carrier is varied according to the source baseband data signal. For binary phase shift keying (BPSK) the phase shift is 180 degrees. BPSK is like FSK a simple form of digital modulation. BPSK can be generated using a balanced mixer, and the two signalling states is 180 degrees apart as shown in figure 2-9.

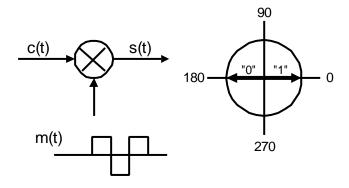


Figure 2-9: PSK modulation and signalling states

The carrier signal c(t) is applied to the mixer. The modulating signal m(t) is bipolar square wave data and output signal s(t) is the PSK modulated carrier. The "1" and the "0" is the two signalling states spaced 180 degrees apart. Figure 2-10 shows the PSK waveform.

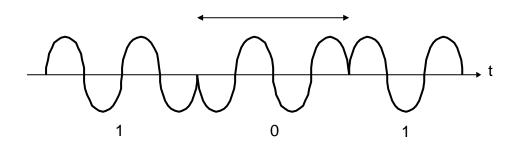


Figure 2-10: Phase Shift Keying Waveform

Abrupt phase changes form 0 degree to 180 degrees, when the data shifts from "1" to "0", gives a wide spectrum. To narrow the spectrum the baseband digital signal is low-pass filtered. In many applications the cosine rolloff filter is used. The filtering slows the phase change and narrows the spectrum.

In BPSK the amplitude goes through the origin (shown in figure 2-9) when the phase shifts between 0 degree and 180 degrees. This means for some time the amplitude is zero. To follow the amplitude fluctuation a linear amplifier is required in the transmitter. If a non-linear amplifier is used the amplitude fluctuation will be distorted and the spectrum will grew wider again, that is the effect of the filtering is removed. PSK is a linear type of modulation.

Another popular form of PSK uses four phase states that are spaced 90 degrees apart. Then two bits in the data stream can be coded in the four states and the twice the data will be transmitted in a given bandwidth. With half the bandwidth the same amount of data can be transmitted as with BPSK. The performance (BER) is equal due to the noise energy in the receiver is half when the bandwidth of the receiver is reduced to one half. The four state PSK is called Quadrature Phase Shift Keying, QPSK, and is more efficient measured in spectrum usage than BPSK.

It is possible to decrease the phase spacing from 90 degrees as in QPSK. This is called mPSK where m is the number of bits per baud. For example a phase states spaced 45 degrees apart gives eight possible constellation points in the IQ plane. Three data bits can be transmitted per phase state. The noise margin in the receiver demodulator will suffer due to the closer phase state spacing. As m increases the noise margin decreases and for wireless applications m greater than eight is rarely found.

After transmission through a channel that only adds white noise to the wanted signal, refereed to as the additive white Gaussian noise (AWGN) channel, BPSK is optimal with respect to the bit energy to noise power density ratio E_b/N_0 . BPSK is therefore used as reference for other types of digital modulation. The bandwidth efficiency is inadequate for terrestrial applications while it is used in deep space satellite communication where extremely weak signals are to be detected.

Quadrature Amplitude Modulation (QAM)

In wireless digital systems where more than three bits per symbol shall be transmitted QAM is often used. In QAM both the amplitude and phase of the carrier is varied according to the baseband data signal. The constellation diagram of 16QAM, in which four bits per baud can be sent, is shown in figure 2-11.

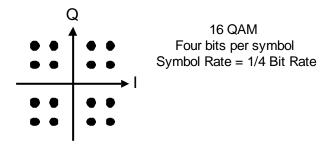


Figure 2-11: 16QAM Constellation Diagram

This modulation format produces a more spectrally efficient transmission than BPSK, QPSK and 8PSK. Note that QPSK is equal to 4QAM.

Offset Quadrature Phase Shift Keying (OQPSK)

Bandlimited PSK needs linear amplifiers which is not power efficient. Other forms of PSK with less amplitude fluctuations for class C amplification are developed.

The first variation is OQPSK. In QPSK the I- and Q-bitstreams are switched simultaneously. In OQPSK the I and Q bitstreams are offset with one bit period (one half of the symbol period). This is shown in figure 2-12.

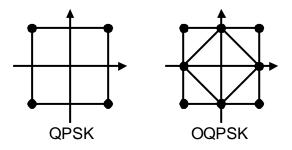


Figure 2-12: Constellation diagram QPSK and OQPSK

Since the transitions of I and Q are offset at any given time only one of the two bitstreams changes state. This creates a different constellation. The signal trajectories are modified by the offset of the I and Q bitstreams so that the carrier amplitude does not go through or near the origin (the centre of the constellation). The reduced amplitude variation, about 3 dB for OQPSK versus 30 to 40 dB for QPSK, allows less linear and more efficient power amplifiers to be used.

p/4 Differential Quadrature Phase Shift Keying (p/4DQPSK)

Another variation of PSK is differential modulation as used in DQPSK. In DPSK the information is carried by the transitions between states and not by the absolute state as in PSK. In some types of DPSK there are restrictions on allowable transitions. One of this modes is $\pi/4$ DQPSK where the carrier trajectory does not go through the origin. The $\pi/4$ DQPSK is used in many applications of one is TETRA (Trans European Trunked Radio).

The $\pi/4DQPSK$ modulation format uses a 45 degree ($\pi/4$) phase shift. The data is encoded in the magnitude and the direction of the phase shift, and is not in the absolute position in the signal constellation. The transmitter design is simplified when the carrier trajectory does not go through the origin. The $\pi/4DQPSK$ with root raised cosine filtering of the baseband signal has high spectral efficiency.

Gaussian Minimum Shift Keying (GMSK)

GMSK is constant envelope modulation. GSM uses a variation of constant envelope modulation called 0.3 GMSK.

In constant envelope modulation the amplitude of the carrier is constant regardless of the changes in the modulating signal. It is a power efficient method that allows class C amplifiers to be used without degradation of the spectral occupancy of the transmitted signal. Constant envelope modulation is not as bandwidth efficient as linear ones like BPSK and QPSK, and it is not suitable in applications where bandwidth efficiency is more important than power efficiency.

MSK is a special type of FSK where the peak-to-peak frequency deviation is equal to one half the baud rate.

Frequency modulation and phase modulation are closely related. A static frequency shift of +1 Hz is equal to a constantly advancing phase shift of 360 degrees per second (2π radians/sec) compared to the unshifted carrier. In MSK a phase shift of 90 degree ($\pi/2$) is used and two signalling states will be orthogonal to each other. The deviation must be accurate in order to generate repeatable 90 degree phase shifts and is suitable to be carried out with an IQ modulator. One common method to encode the data bits is to let +90 degree phase shift be the logical one and -90 degree phase shift be the logical zero. Phase shifts gradually from completely aligned in the centre of one bit to that completely aligned in the centre of the next bit. The angular speed of the vector (amplitude of the signal) in the IQ plane is 90 degree/ $T_{\rm bit}$ where $T_{\rm bit}$ is duration of one bit period. After turning through the angle the vector does not remain stationary in the position reached but continues in the same direction if the next bit is a one or turns in the opposite direction if the next bit is a zero. The result of MSK is a constant envelope signal with gradually changing phase. Figure 2-13 shows the relationships between data signals and phase in a system employing MSK modulation.

GMSK relates to the filtering of the data before modulation. The filter passband has a Gaussian shape (more information in the next section Optimum Modem Filters). In GMSK the term BT_{Bit} is used to describe the efficiency of the filter. BT_{Bit} normalises the filter bandwidth (3 dB bandwidth) to the bit frequency.

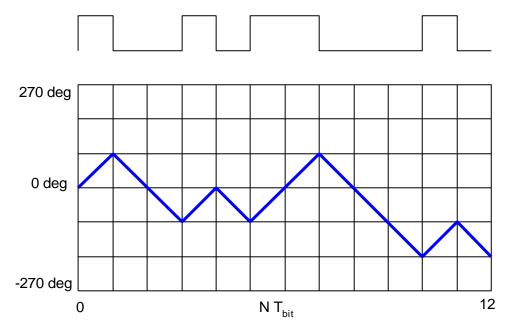


Figure 2-13: Phase versus data in MSK modulation

The Gaussian filtering cannot be carried out using analogue techniques. Figure 2-14 shows a digital implementation of GMSK modulation.

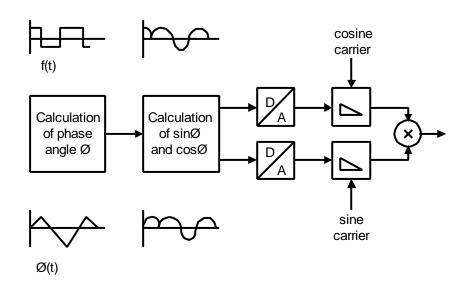


Figure 2-14: IQ modulator for GMSK

Table 2-1 summarises the main parameters for MSK modulation used in GSM.

Modulation index (h)	Frequency deviation (D f)	Phase shift over bit duration (T _{bit})	Correlation factor (p)	Euclidean distance (D)
0.5	$1/4 T_{bit}$	90 degree $(\pi/2)$	0	$\sqrt{2} E_{\rm bit}$

Table 2-1: MSK parameters at a glance

Optimum Filters

When a RF carrier is modulated with a square wave data signal a wide spectrum is produced. Filtering allows the transmitted bandwidth to be significantly reduced without loosing the content of the data. A key question that arises is how to determine the optimum frequency response of a system.

There are different types of filters. The most common types are

- Gaussian filters
- Raised cosine
- Square root raised cosine

Any fast transition in a signal requires a wide bandwidth. Filters do the function of slowing down the transition and then to reduce the bandwidth. Some types of filters cause the trajectory of the signal between states to overshoot which requires more power to transmit than the actual symbol itself. The carrier power cannot be limited (clipped) without causing the spectrum to spread out again.

Filtering can also create Inter Symbol Interference (ISI). Filtering in the frequency domain causes the energy to spread out in the time domain. Figure 2-15 shows a simple example of a first order low pass filter fed with a square wave signal.

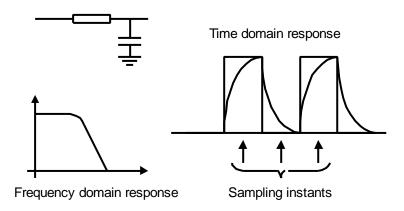


Figure 2-15: Simple filtering in frequency and time domain.

Consider the sequence one-zero, energy from the one blurs into the sampling instant of the zero. This effect makes the margins for bit decoding in the receiver less.

The place where it is easiest to control the frequency response is at baseband. This is because the filter function to the channel response is most obvious, in fact it is the same function. Filtering at the RF carrier is theoretically possible but in practice difficult to control precisely. (At higher carrier frequencies, VHF and UHF, impossible).

Gaussian Filters

Gaussian filters will have small blurring from the previous signalling state, and a trade-off has to be done between bandwidth efficiency and the inter symbol interference due to the blurring that can be tolerated. The Gaussian filter has a Gaussian shape in both the frequency domain and the time domain. The name Gaussian relates to the equality between the filter response and the shape of the Gaussian normal distribution curve.

Gaussian filters are used in GSM, which uses constant envelope GMSK modulation, due to the advantages in carrier power and bandwidth efficiency.

Raised Cosine Filters

One family of filters used in many applications using linear modulation methods like PSK, QPSK and QAM is the raised cosine one. The filter rolloff is determined with factor α which varies from zero to one. An α of zero means a perfect brick-wall filter, i.e. a filter that passes frequencies below the corner frequency unattenuated, and attenuates frequencies above corner frequency hundred percent. The brick-wall filter is impossible to implement and in practice for RF transmission α between 0.2 and 0.5 is used. In figure 2-16 roll-off of the raised cosine filter for α =0.3 (f_0), α =0.5 (f_1) and α =1.0 (f_2) with data baud rate 8k baud is shown.

Note that the responses pass through one half at half the baud rate. This is an important property of the raised cosine filter. The occupied bandwidth of a system is calculated as

$$occupied\ bandwidth = symbol\ ratex(1 + a)$$

As noted earlier in this chapter, filtering in the frequency domain causes signal energy to be spread out in the time domain; the filter rings in the time domain. The raised cosine filter has the important property that the ringing passes through zero at the symbol rate. Figure 2-17 shows the impulse response of the raised cosine filter in the time domain.

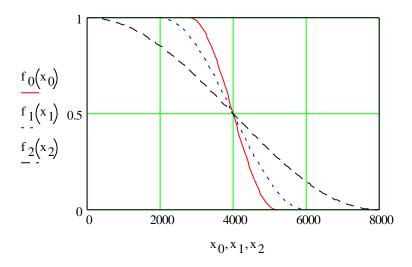
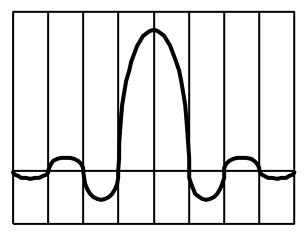


Figure 2-16: Raised cosine filter roll-off



Time - each division equals on bit period

Figure 2-17: Raised cosine filter time domain response

The ringing from previous data bits adds energy to the current one, but when the sampling instant is chosen at time when the ringing passes through zero it has no impact of the decoding.

Filtering causes the amplitude to change and for some time instants to overshoot. The overshoot requires more power to be transmitted and the transmitter power amplifier must be capable of transmitting the effect at the overshoots without distortion. This effect is not wanted from a cost and efficiency point of view. Figure 2-18 shows examples of signal trajectories between the signalling points for different α .

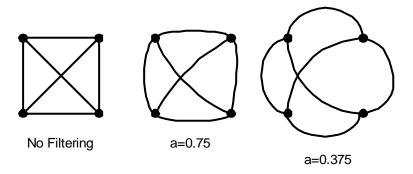


Figure 2-18: Effect of different filter bandwidth

No filtering means an α of infinity which requires infinite bandwidth. The maximum peak power is equal to the power in the symbol states, and no extra power is required due to the filtering. With $\alpha = 0.375$ the power in the trajectories between the symbol states exceeds that in the symbol state itself. It is a trade-off between bandwidth and power efficiency.

It has been shown that the optimum response in a digital transmission system is obtained when equal filters are used in the transmitter and receiver. The total response is the two responses multiplied with each other, and then using the square root of the response in the transmitter and receiver gives the total response as desired. The filter to be used in the transmitter is then the square root raised cosine.

References

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